ON STABILIZED FINITE ELEMENT METHODS FOR ADVECTION-DIFFUSION-REACTION PROBLEMS WITH VARYING TIME SCALES

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Many engineering and scientific applications require a detailed analysis of strongly coupled continuum transport and chemical reaction physics with multiple time and length scales. These systems arecharacterized by the co-existence of transport time scales (advection and diffusion) and very short timescales for non-equilibrium chemical reactions. The stable and accurate finite element modeling of the reaction phase may require time steps which are orders of magnitude smaller than those normally required in the flow solver. For this reason in a typical reacting flow application an implicit time stepping scheme is often preferred to allow efficient and accurate treatment of this stiff behavior. Some of the most effective algorithms for treating such problems can be defined through a process wherein spatial and temporal discretizations are separated.

In this talk we investigate the impact of this very fine time scale resolution when an implicit finite difference integration in time is coupled with streamline upwind Petrov-Galerkin discretization in the spatial variable. It is known that implicit time integration at a sufficiently small time step converges to a singularly perturbed elliptic problem. The use of consistent spatial stabilization leads to the occurrence of additional couplings between the spatial and temporal components of the scheme. These couplings can be associated with anti-dissipative terms and as a result, they can have a potentially destabilizing effect on the fully discrete algorithm. We provide some preliminary theoretical and computational results that illustrate the onset of spurious oscillations at very fine time steps. Some possible ways to avoid instabilities are also considered, including space-time formulations.